

Spin Hall Effect and Origins of Nonlocal Resistance in Adatom-Decorated Graphene

D. Van Tuan,^{1,2} J. M. Marmolejo-Tejada,^{3,4} X. Waintal,⁵ B. K. Nikolić,^{3,*} S. O. Valenzuela,^{1,6} and S. Roche^{1,6,†}

¹*Catalan Institute of Nanoscience and Nanotechnology (ICN2),
CSIC and The Barcelona Institute of Science and Technology,
Campus UAB, Bellaterra, 08193 Barcelona, Spain*

²*Department of Electrical and Computer Engineering,
University of Rochester, Rochester, NY 14627, USA*

³*Department of Physics and Astronomy, University of Delaware, Newark, DE 19716-2570, USA*

⁴*School of Electrical and Electronics Engineering,
Universidad del Valle, Cali, AA 25360, Colombia*

⁵*CEA-INAC/UJF Grenoble 1, SPSMS UMR-E 9001, Grenoble F-38054, France*

⁶*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*
(Dated: September 29, 2016)

Recent experiments reporting unexpectedly large spin Hall effect (SHE) in graphene decorated with adatoms have raised a fierce controversy. We apply numerically exact Kubo and Landauer-Büttiker formulas to realistic models of gold-decorated disordered graphene (including adatom clustering) to obtain the spin Hall conductivity and spin Hall angle, as well as the nonlocal resistance as a quantity accessible to experiments. Large spin Hall angles of ~ 0.1 are obtained at zero-temperature, but their dependence on adatom clustering differs from the predictions of semiclassical transport theories. Furthermore, we find multiple background contributions to the nonlocal resistance, some of which are unrelated to SHE or any other spin-dependent origin, as well as a strong suppression of SHE at room temperature. This motivates us to design a multiterminal graphene geometry which suppresses these background contributions and could, therefore, quantify the upper limit for spin current generation in two-dimensional materials.

PACS numbers: 72.80.Vp, 73.63.-b, 73.22.Pr, 72.15.Lh, 61.48.Gh

Introduction.—Over the past decade, the spin Hall effect (SHE) has evolved rapidly from an obscure theoretical prediction to a major resource for spintronics [1, 2]. In the direct SHE, injection of conventional unpolarized charge current into a material with extrinsic (due to impurities) or intrinsic (due to band structure) spin-orbit coupling (SOC) generates pure spin current in the direction transverse to charge current. Although SHE was first observed only a decade ago [3], it is already ubiquitous within spintronics as standard pure spin-current generator and detector [1, 2]. The spin Hall angle θ_{SH} , as the ratio of generated spin Hall current and injected charge current, is the figure of merit for charge-to-spin conversion efficiency. To date, measured values of θ_{SH} range from $\sim 10^{-4}$ in semiconductors to ~ 0.1 in metals like β -Ta and β -W [2].

Concurrently, the discovery of graphene [4] has ignited a considerable amount of activity, owing to its unique electronic properties and versatility for practical applications, including possible applications in spintronics [5]. The intrinsically small SOC and hyperfine interactions [6] in graphene lead to spin relaxation lengths reaching several tens of micrometers at room temperature [7–10], but simultaneously making pristine graphene inactive for SHE [5]. On the other hand, recent nonlocal transport measurements on graphene decorated with heavy adatoms like copper, gold and silver have extracted exceptionally large values for $\theta_{\text{SH}} \sim 0.2$ [11]. These reports follow prior experiments on weakly hydrogenated

graphene, which showed surprisingly similar results [12] despite using light adatoms like hydrogen. The large values of θ_{SH} observed in both types of experiments have been supported by semiclassical transport theories [13, 14].

The very recent experiments [15, 16] aiming to reproduce these results have indeed confirmed large nonlocal transport signal near the charge neutral point (CNP) of graphene which, however, appears to be disconnected from SHE physics or any other spin-related mechanism. For example, Wang and coworkers [15] reported that Au- or Ir-decorated graphene exhibits no signature of SHE and relate the large nonlocal resistance R_{NL} to the formation of neutral Hall currents. Kaverzin and van Wees [16] found large R_{NL} in hydrogenated graphene which was insensitive to an applied in-plane magnetic field. These authors [16] exclude valley Hall effect and long-range chargeless valley currents [17] as mediating such R_{NL} , given the absence of both its temperature dependence and broken inversion symmetry, and conclude that a non-trivial and unknown phenomenon is at play.

The presently available theories for θ_{SH} [13] or R_{NL} [18] offer little guidance on how to resolve these controversies since they utilize semiclassical approaches to charge transport and spin relaxation which are known to break down [19, 20] near CNP. Moreover, while Kubo formula [21] offers fully quantum-mechanical treatment that can in principle capture all relevant effects, its standard analytic evaluation neglects [22] terms (such as those

corresponding to skew-scattering from pairs of closely spaced impurities [23]) in the perturbative expansion in disorder strength which can become crucial for clusters of adatoms. Finally, the impact of unavoidable adatom clustering [24] on θ_{SH} is an open and important question, since adatom segregation has been shown to strongly affect spin transport properties [25, 26].

In this Letter, the spin Hall angle in graphene decorated with Au-adatoms is computed by using two different *numerically exact* quantum transport methodologies—the real-space Kubo formula and the multiterminal Landauer-Büttiker (LB) formula [27]. At zero temperature, both methods yield $\theta_{\text{SH}} \sim 0.1\text{--}0.3$ for the same Au-adatom concentration n_i . However, those values require rather large $n_i \gtrsim 10\%$ and drop significantly when temperature and adatom clustering are taken into account.

Furthermore, the LB formula applied to six-terminal graphene geometry in Fig. 1 reveals large background contributions to R_{NL} even when *SOC is artificially turned off*. They are, therefore, unrelated to SHE physics, and are also unrelated to trivial Ohmic contribution due to classical current paths [16, 18]. We show that their sign [28, 29] and scaling with the channel length L makes it possible to understand their origin. This allows us to propose a novel six-terminal graphene setup—see Fig. S7 in the Supplemental Material (SM) [30]—where such background contributions can be eliminated in order to study purely SHE-driven R_{NL} signal.

Hamiltonian model for Au-decorated graphene.—When an adatom like gold, thallium or indium is absorbed onto graphene surface, it resides in the center of graphene carbon rings where it can enhance the intrinsic SOC or induce Rashba SOC due to the broken inversion symmetry [34]. The minimal (with single π orbital per site) effective tight-binding model for graphene with such adatoms is given by

$$\begin{aligned} \mathcal{H} = & -\gamma_0 \sum_{\langle ij \rangle} c_i^\dagger c_j + \frac{2i}{\sqrt{3}} V_I \sum_{\langle\langle ij \rangle\rangle \in \mathcal{R}} c_i^\dagger \vec{s} \cdot (\vec{d}_{kj} \times \vec{d}_{ik}) c_j \\ & + iV_R \sum_{\langle ij \rangle \in \mathcal{R}} c_i^\dagger \vec{z} \cdot (\vec{s} \times \vec{d}_{ij}) c_j - \mu \sum_{i \in \mathcal{R}} c_i^\dagger c_i. \end{aligned} \quad (1)$$

The first term is the nearest neighbor hopping term with $\gamma_0 = 2.7$ eV. The second term is the next-nearest neighbor hopping term which accounts for the local intrinsic SOC enhancement by adatoms residing on the set of hexagons \mathcal{R} . The unit vector \vec{d}_{kj} points from atom j to atom k , with atom k standing in between i and j , and $\vec{s} = (s_x, s_y, s_z)$ is the vector of the Pauli matrices. The third term is the nearest neighbor hopping term describing the Rashba SOC which explicitly violates $\vec{z} \rightarrow -\vec{z}$ symmetry. The last term is the on-site potential μ on carbon atoms in the hexagons hosting adatoms, which simulates charge modulation induced locally around the adatom [34]. The Hamiltonian in Eq. (1) has been em-

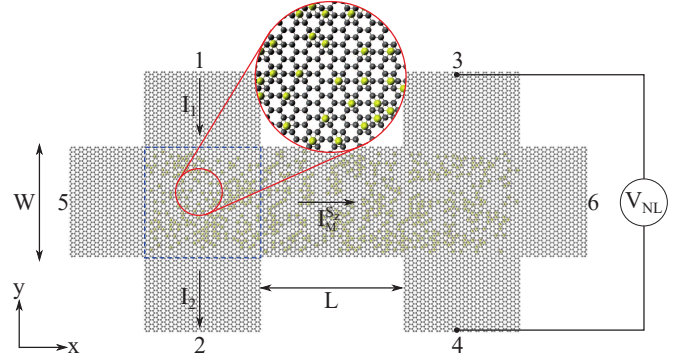


FIG. 1. (Color online) Schematic view of a six-terminal graphene employed to compute the nonlocal resistance $R_{\text{NL}} = V_{\text{NL}}/I_1$ and the spin Hall angle $\theta_{\text{SH}} = I_5^z/I_1$. For nonlocal transport, the injected transverse charge current between leads 1 and 2 generates the longitudinal spin current I_5^z in lead 5, as well as the mediative spin current I_M^z whose conversion into the voltage drop $V_{\text{NL}} = V_3 - V_4$ between leads 3 and 4 generates R_{NL} . The dashed region illustrates the sample of size $400 \text{ nm} \times 400 \text{ nm}$, with periodic boundary conditions, used for calculations of Kubo conductivities. Zoom shows carbon atoms (black circles) and Au adatoms (yellow circles).

ployed to study spin dynamics in graphene decorated with Au-adatoms [19], and here we use the same parameters $V_I = 0.007\gamma_0$, $V_R = 0.0165\gamma_0$ and $\mu = 0.1\gamma_0$ fitted to first-principles calculations [34].

Figure 1 shows the geometry used for the calculations of bulk Kubo conductivities and multiterminal charge and spin currents. The calculations of θ_{SH} with the Kubo formula are performed using a graphene flake of the size $400 \text{ nm} \times 400 \text{ nm}$ enclosed in dashed square with periodic boundary conditions. For LB calculations we consider full six-terminal geometry in Fig. 1, where the central region with edges of armchair type, width $W = 50 \text{ nm}$ (composed of $3n + 2$ dimer lines, so that its electronic structure resembles that of large-area graphene [20]) and variable distance L between the pair of leads 1 and 2 and the pair of leads 3 and 4 is attached to two armchair longitudinal leads and four transverse leads with zigzag edges and of width $W = 50 \text{ nm}$. Akin to experimental procedure [11, 12, 15, 16, 35], injecting unpolarized charge current I_1 into this measurement geometry induces $R_{\text{NL}} = (V_3 - V_4)/I_1$ and $\theta_{\text{SH}} = I_5^z/I_1$.

Real-space Kubo formula for spin Hall conductivity.—The Kubo formula for spin Hall conductivity σ_{SH} reads [2]

$$\sigma_{\text{SH}} = \frac{e\hbar}{\Omega} \sum_{m,n} \frac{f(E_m) - f(E_n)}{E_m - E_n} \frac{\text{Im} [\langle m | J_x^z | n \rangle \langle n | v_y | m \rangle]}{E_m - E_n + i\eta}, \quad (2)$$

where v_x is the velocity operator and $J_x^z = \frac{\hbar}{4} \{s_z, v_x\}$ is the spin current operator. The numerical evaluation of Eq. (2) is usually made by finding the whole spectrum E_m and the full set of eigenvectors $\{|m\rangle\}$ of \mathcal{H} , which is computationally expensive task. Here we develop al-

ternative and efficient real-space formalism by re-writing σ_{sH} as

$$\sigma_{\text{sH}} = \frac{e\hbar}{\Omega} \int dx dy \frac{f(x) - f(y)}{(x-y)^2 + \eta^2} j(x, y), \quad (3)$$

with $j(x, y) = \sum_{m,n} \text{Im} \{ \langle m | J_x^z | n \rangle \langle n | v_y | m \rangle \} \delta(x - E_m) \delta(y - E_n)$. This can be calculated by rescaling \mathcal{H}, x, y and E into the interval $[-1, 1]$ (the corresponding variables are h, \hat{x}, \hat{y} and ϵ , respectively) and by expanding $j(x, y)$ into Chebyshev polynomials $T_m(\hat{x})$ as $j(x, y) = \sum_{m,n}^M [4\mu_{mn} g_m g_n T_m(\hat{x}) T_n(\hat{y})] / [(1 + \delta_{m,0})(1 + \delta_{n,0})\pi^2 \sqrt{(1 - \hat{x}^2)(1 - \hat{y}^2)}]$, where $\mu_{mn} = \text{Im} \{ \text{Tr} [J_x^z T_n(h) v_y T_m(h)] \} / \Delta E^2$ and ΔE is half the bandwidth [36]. Here g_m is the filter, Jackson kernel, that minimizes the Gibbs oscillations arising in truncating the series to finite order M [36]. The trace in μ_{mn} is computed by averaging [37] over a small number $r \ll N$ of random phase vectors $|\varphi\rangle$, with N being the number of carbon atoms considered in the sample. Hereafter $M = 1500$ ($= 6000$) for σ_{sH} (σ_{xx}), $r = 1$ and $N = 4 \times 10^6$. Similar methods have been developed for the longitudinal conductivity σ_{xx} [37], Hall conductivity σ_{xy} [38, 39] and spin Hall conductivity σ_{sH} [40]. The method is validated by comparing our numerically evaluated σ_{sH} with analytic results [41] for clean graphene with homogeneous Rashba or intrinsic SOC [30].

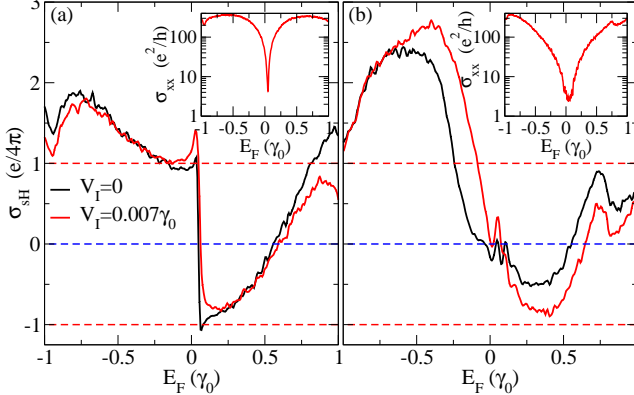


FIG. 2. (Color online): Spin Hall σ_{sH} (main frame) and longitudinal σ_{xx} (insets) conductivities for the two cases of $n_i = 15\%$ Au-adatom distributions: (a) scattered and (b) clustered, where Au islands have varying radius $\in [1, 3]$ nm. In both cases the effect of the presence (red lines, $V_I = 0.007\gamma_0$) or absence (black lines, $V_I = 0$) of the enhanced intrinsic SOC within the hexagons hosting adatoms is also shown. All results are averaged over 400 disorder configurations.

Spin Hall angle for different adatom distributions.— Figure 2 shows σ_{sH} for $n_i = 15\%$ of Au adatoms distributed in scattered [Fig. 2(a)] or clustered fashion [Fig. 2(b)], where clusters are randomly distributed islands of radius $\in [1, 3]$ nm. Although the random distribution of Au adatoms and the Rashba SOC associated with them

induce scattering [$\mu = 0.1\gamma_0$ in Eq. (1)], the dependence of σ_{sH} on the Fermi energy E_F in the absence of intrinsic SOC is reminiscent of a step behavior obtained for a homogeneous Rashba SOC [30], with $\sigma_{\text{sH}} \simeq \pm e/4\pi$ near CNP. Adding a small intrinsic SOC, $V_I = 0.007\gamma_0 \ll V_R$, slightly changes the absolute value of σ_{sH} but preserves the step behavior. In contrast, the clustered distribution of Au adatoms suppresses the step behavior and smooths out the shape of σ_{sH} close to CNP. The effect of intrinsic SOC is more pronounced for the clustered distribution with a more significant enhancement of σ_{sH} on both electron and hole side.

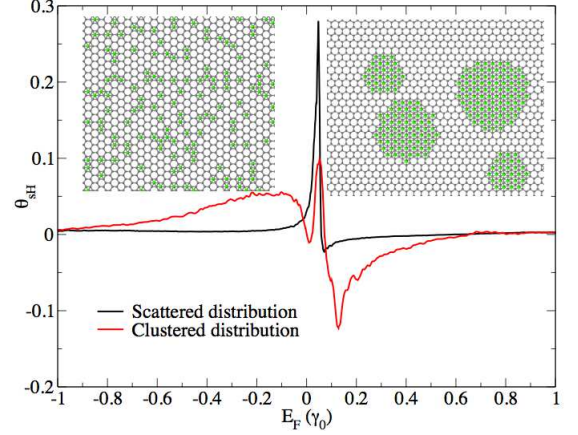


FIG. 3. (Color online) Spin Hall angle $\theta_{\text{sH}} = \sigma_{\text{sH}}/\sigma_{xx}$ corresponding to Fig. 2 for scattered (black) and clustered (red) distributions of Au adatoms, which are illustrated in the insets.

The spin Hall angle $\theta_{\text{sH}} = \sigma_{\text{sH}}/\sigma_{xx}$ requires additional calculation of the longitudinal conductivity σ_{xx} , which is performed using a real-space Kubo formula [30]. Figure 2 (insets) shows σ_{xx} for both cases. Comparable values of σ_{xx} are obtained at CNP, but for the scattered case σ_{xx} increases with energy faster than for the clustered case. Figure 3 shows θ_{sH} for $n_i = 15\%$ of Au adatoms, which are distributed homogeneously (black lines) or in clusters (red line). Remarkably, the values of θ_{sH} shown in Fig. 3 are very large ~ 0.1 close to CNP, which is similar to experimentally reported values [11]. At the CNP, a threefold decrease in θ_{sH} is obtained when adatoms are clustered into islands with small radius. This conclusion seems to differ from the semiclassical transport predictions where θ_{sH} increases with the radius of adatom clusters [13], although a strict comparison would require to treat a system consisting of identical islands. At higher energies, we observe a sizable θ_{sH} for clustered adatoms, which contrasts with vanishingly small values for the scattered geometry. We finally extrapolate that for $n_i = 2-3\%$ (as estimated in experiments [11]), θ_{sH} should range between 0.01–0.1 (see SM [30]). We stress, however, that our calculations represent an upper limit for experimental situations. There, the increase of the

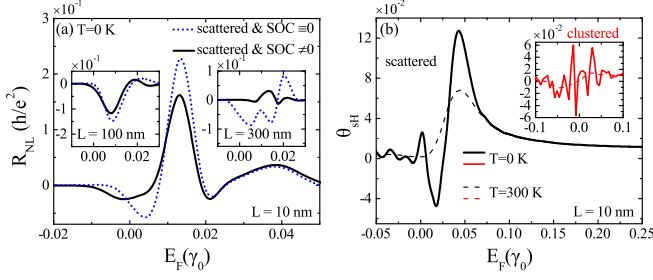


FIG. 4. (Color online) (a) Nonlocal resistances for six-terminal graphene in Fig. 1 with $n_i = 15\%$ of scattered Au adatoms, fixed channel width $W = 50$ nm and several channel lengths: $L = 10$ nm (main frame); $L = 100$ nm (left inset); and $L = 300$ nm (right inset). Dotted lines plot R_{NL} when all SOC terms in Eq. (1) are switched off ($\text{SOC} \equiv 0 \Leftrightarrow V_I = V_R = 0$). (b) Spin Hall angle, obtained from LB formula calculations, for the same concentration of Au adatoms which are scattered (main frame) or clustered (inset). All curves are averaged over 10 disorder configurations.

cluster size and the finite temperature can significantly decrease θ_{SH} below 0.01 [30].

It is also instructive to compare the results in Fig. 2 to the case of heavier adatoms like thallium (Tl), which locally and substantially enhance the intrinsic SOC while generating negligible Rashba SOC [34]. A crossover from quantum SHE to conventional SHE has been predicted [26] when the distribution of Tl adatoms is changed from scattered to clustered. Figure S2 in the SM [30] shows that SHE due to clustered Tl adatoms is characterized by larger θ_{SH} than in the case of either scattered or clustered Au adatoms.

Nonlocal resistance and spin Hall angle in multiterminal graphene.—In the SHE experiments [11, 12], multiterminal graphene devices are employed to measure R_{NL} , as illustrated in Fig. 1. In such a circuit, a charge current I_1 injected from lead 1 towards lead 2 generates the nonlocal resistance $R_{NL} = (V_3 - V_4)/I_1$ at the Fermi energy E_F sufficiently close to CNP. The appearance of nonzero R_{NL} , due to a SHE-driven mechanism, is explained by charge current I_1 inducing mediative spin current $I_M^{S_z}$ in the first crossbar in Fig. 1 flowing in the direction $5 \rightarrow 6$, which is subsequently converted into the nonlocal voltage $V_{NL} = V_3 - V_4$ by the inverse SHE in the second crossbar. We calculate the total charge I_p and spin $I_p^{S_z}$ currents and voltages V_p in leads $p = 2-6$ in response to injected charge current I_1 using the multiterminal LB formula [27], as implemented in KWANT software package [30, 42].

The spin Hall angle—defined as $\theta_{\text{SH}} = I_5^{S_z}/I_1$ —is shown in Fig. 4(b) where we confirm large values obtained from the Kubo formula, as well as the detrimental effect of clustering of Au adatoms. While both Kubo and LB formula calculations predict $\theta_{\text{SH}} \simeq 0.1$ close to the CNP, thermal broadening effects included in LB formula calcu-

lations can reduce θ_{SH} by up to one order of magnitude [see Fig. 4(b)]. By comparing Fig. 4(b) with Fig. S4 of the SM [30], we find that the hypothetical case of homogeneous Rashba SOC, due to Au adatoms covering every hexagon in Fig. 1, generates SHE akin to the intrinsic one in finite-size two-dimensional electron gases [43, 44]. Its θ_{SH} exhibits wider peak (centered at $E_F = 0.3\gamma_0$ due to doping of graphene by Au adatoms) of smaller magnitude than in the case of randomly scattered Au adatoms. Thus, adatom induced resonant scattering [13] plays an important role in generating large extrinsic SHE.

Figure 4(a) shows R_{NL} as a function of energy and for various channel lengths L . Most notably, we find a nonzero R_{NL} even when all SOC terms are switched off ($V_R = V_I = 0$) in Eq. (1), while keeping random on-site potential $\mu \neq 0$ due to Au adatoms unchanged. Furthermore, we find a complex sign change of R_{NL} in Fig. 4(a) with increasing channel length from $L = 10$ nm to $L = 300$ nm, which suggests the following interpretation. The total R_{NL} can have four contributions $R_{NL} = R_{NL}^{\text{SHE}} + R_{NL}^{\text{Ohm}} + R_{NL}^{\text{qb}} + R_{NL}^{\text{pd}}$, assuming they are additive after disorder averaging. For unpolarized charge current injected from lead 1 (i.e., electrons injected from lead 2): R_{NL}^{SHE} generated by combined direct and inverse SHE has a positive sign; trivial Ohmic contribution R_{NL}^{Ohm} due to classical diffusive charge transport [16, 18] has a positive sign; R_{NL}^{qb} is the negative quasiballistic contribution arising due to direct transmission $T_{32} \neq 0$ from lead 2 to lead 3 (see Fig. S6 in SM [30]), as observed previously in SHE experiments on multiterminal gold devices [28]; finally, R_{NL}^{pd} is a positive contribution specific to Dirac materials where evanescent wavefunctions generate pseudodiffusive transport [45] close to CNP characterized by two-terminal conductance scaling as $G \propto 1/L$ even in perfectly clean samples as long as their geometry satisfies $W > L$ (see Fig. S5 in SM [30]).

Thus, in device with $W > L$, such as $W = 50$ nm and $L = 10$ nm in the main frame of Fig. 4(a), the positive sign R_{NL} is dominated by R_{NL}^{pd} , which can be larger than in the case of perfectly clean graphene in Fig. S5 of SM [30] due to scattering from impurities (of uniform strength) at CNP [46]. The negative sign of R_{NL} in the two insets in Fig. 4(a) in the absence of SOC and for $L > W$ suggests that R_{NL}^{Ohm} can be safely neglected in our samples due to small n_i —we estimate the mean free path $\ell = 300-400$ nm for $n_i = 15\%$, so that when diffusive transport regime sets in for $\ell > L$ the Ohmic contribution $R_{NL}^{\text{Ohm}} \propto \exp(-\pi L/W)$ [16, 18] is already negligible due to $L/W \gg 1$. Therefore, for $L > W$ the main competition is between R_{NL}^{qb} with negative sign and R_{NL}^{SHE} with positive sign, as found in the two insets of Fig. 4(a). The existence of background contributions to R_{NL} that do not originate from SHE, and can be even larger than R_{NL}^{SHE} , could explain insensitivity of the total R_{NL} to the applied external in-plane magnetic field observed in some

experiments [15, 16].

The difficulty in clarifying the dominant contribution to R_{NL} could be resolved by detecting its sign change as a function of the channel length L in Fig. 1. An alternative is to design a setup where $R_{\text{NL}}^{\text{Ohm}}$, $R_{\text{NL}}^{\text{qb}}$, and $R_{\text{NL}}^{\text{pd}}$ are negligible so that $R_{\text{NL}}^{\text{SHE}}$ can be isolated. We propose such setup in Fig. S7 of the SM [30] where adatoms are removed in the channel. When such channel is sufficiently long, $R_{\text{NL}}^{\text{pd}} = 0$ due to $L > W$ and $R_{\text{NL}}^{\text{Ohm}}, R_{\text{NL}}^{\text{qb}} \rightarrow 0$ due to the absence of impurity scattering in the channel, so that mediative spin current $I_M^{S_z}$ generated by direct SHE in the first crossbar arrives conserved [44] at the second crossbar where it is converted into V_{NL} by the inverse SHE. Indeed, Fig. S8 of the SM [30] demonstrates that R_{NL} and θ_{SH} in this setup are unambiguously related since they both display sharp peak at virtually the same E_F very close to CNP. The upper limit on $R_{\text{NL}}^{\text{SHE}}$ obtained by this procedure in Fig. S8(a) of the SM [30] is still smaller than the absolute values of other non-SHE contributions to R_{NL} in Fig. 4(a).

A. W. Cummings and J. H. Garcia are acknowledged for critical reading of the manuscript. This work was supported by European Union Seventh Framework Programme under grant agreement 604391 Graphene Flagship. S. R. acknowledges the Spanish Ministry of Economy and Competitiveness for funding (MAT2012-33911), the Secretaria de Universidades e Investigacion del Departamento de Economia y Conocimiento de la Generalidad de Catalunya and the Severo Ochoa Program (MINECO SEV-2013-0295). J. M. M.-T. was supported as Fulbright Scholar and by Colciencias (Departamento Administrativo de Ciencia, Tecnologia e Innovacion) of Colombia. B. K. N. was supported by NSF Grant No. ECCS 1509094, and is grateful for the hospitality of Centre de Physique Théorique de Grenoble-Alpes where part of this work was done. S. O. V. acknowledges support from the European Research Council under Grant Agreement No. 308023 SPINBOUND. The supercomputing time was provided by XSEDE, which is supported by NSF Grant No. ACI-1053575.

* bnikolic@udel.edu

† stephan.roche@icn2.cat

- [1] G. Vignale, J. Supercond. Nov. Magn. **23**, 3 (2010).
- [2] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Rev. Mod. Phys. **87**, 1213 (2015).
- [3] Y. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science **306**, 1910 (2004); J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005); S. O. Valenzuela and M. Tinkham, Nature **442**, 176 (2006); E. Saitoh, M. Ueda, H. Miyajima, and G. Tatara, Appl. Phys. Lett. **88**, 182509 (2006).
- [4] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, Rev. Mod. Phys. **81**, 109 (2009); A. Ferrari *et al.* Nanoscale **7**, 4598 (2015).
- [5] W. Han, R. K. Kawakami, M. Gmitra, and J. Fabian, Nat. Nanotech. **9**, 794 (2014); S. Roche *et al.*, 2D Mater. **2**, 030202 (2015).
- [6] D. Huertas-Hernando, F. Guinea, and A. Brataas, Phys. Rev. B **74**, 155426 (2006); M. Gmitra, S. Konschuh, C. Ertler, C. Ambrosch-Draxl, and J. Fabian, Phys. Rev. B **80**, 235431 (2009); A. H. Castro Neto and F. Guinea, Phys. Rev. Lett. **103**, 026804 (2009); V. K. Dugaev, E. Ya. Sherman, and J. Barnas, Phys. Rev. B **83**, 085306 (2011).
- [7] N. Tombros, C. Jozsa, M. Popinciuc, H. T. Jonkman, and B. J. Van Wees, Nature **448**, 571 (2007).
- [8] K. Pi, W. Han, K. M. McCreary, A. G. Swartz, Y. Li, and R. K. Kawakami, Phys. Rev. Lett. **104**, 187201 (2010).
- [9] B. Dlubak *et al.*, Nat. Phys. **8**, 557 (2012).
- [10] M. H. D. Guimarães, P. J. Zomer, J. Ingla-Aynes, J. C. Brant, N. Tombros, and B. J. van Wees, Phys. Rev. Lett. **113**, 086602 (2014); M. Drögel, F. Volmer, M. Wolter, B. Terres, K. Watanabe, T. Taniguchi, G. Güntherodt, C. Stampfer, and B. Beschoten, Nano Lett. **14**, 6050 (2014); M. Venkata Kamalakar, C. Groenveld, A. Dankert, and S. P. Dash, Nat. Commun. **6**, 6766 (2015); B. Raes, *et al.* Nat. Commun. **7**, 11444 (2016).
- [11] J. Balakrishnan *et al.*, Nat. Commun. **5**, 4748 (2014).
- [12] J. Balakrishnan *et al.*, Nat. Phys. **9**, 284 (2013).
- [13] A. Ferreira, T. G. Rappoport, M. A. Cazalilla, and A. H. Castro Neto, Phys. Rev. Lett. **112**, 066601 (2014).
- [14] H.-Y. Yang, C. Huang, H. Ochoa, and M. A. Cazalilla, Phys. Rev. B **93**, 085418 (2016).
- [15] Y. Wang, X. Cai, J. Reutt-Robey, and M. S. Fuhrer, Phys. Rev. B **92**, 161411 (2015).
- [16] A. A. Kaverzin and B. J. van Wees, Phys. Rev. B **91**, 165412 (2015).
- [17] R. V. Gorbachev *et al.*, Science **346**, 448 (2014).
- [18] D. A. Abanin, A. V. Shytov, L. S. Levitov, and B. I. Halperin, Phys. Rev. B **79**, 035304 (2009).
- [19] D. V. Tuan, F. Ortmann, D. Soriano, S. O. Valenzuela, and S. Roche, Nat. Phys. **10**, 857 (2014); D. V. Tuan, F. Ortmann, A. W. Cummings, D. Soriano, and S. Roche, Sci. Rep. **6**, 21046 (2016); A. W. Cummings and S. Roche, Phys. Rev. Lett. **116**, 106601 (2016).
- [20] C.-L. Chen, C.-R. Chang, and B. K. Nikolić, Phys. Rev. B **85**, 155414 (2012).
- [21] P. Středa, J. Phys. C: Solid State Phys. **15**, L717 (1982).
- [22] N. A. Sinitsyn, J. E. Hill, H. Min, J. Sinova, and A. H. MacDonald, Phys. Rev. Lett. **97**, 106804 (2006).
- [23] I. A. Ado, I. A. Dmitriev, P. M. Ostrovsky, and M. Titov, Phys. Rev. Lett. **117**, 046601 (2016); M. Milletari and A. Ferreira, arXiv:1601.08076; *ibid*, arXiv:1604.03111.
- [24] E. Sutter *et al.*, Surf. Sci. **605**, 1676 (2011).
- [25] K. Pi, K. M. McCreary, W. Bao, W. Han, Y. F. Chiang, Y. Li, S.-W. Tsai, C. N. Lau, and R. K. Kawakami, Phys. Rev. B **80**, 075406 (2009); M. I. Katsnelson, F. Guinea, and A. K. Geim, Phys. Rev. B **79**, 195426 (2009); K. M. McCreary, K. Pi, A. G. Swartz, W. Han, W. Bao, C. N. Lau, F. Guinea, M. I. Katsnelson, and R. K. Kawakami, Phys. Rev. B **81**, 115453 (2010).
- [26] A. Cresti, D. Van Tuan, D. Soriano, A. W. Cummings, and S. Roche, Phys. Rev. Lett. **113**, 246603 (2014).
- [27] M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).
- [28] G. Mihajlović, J. E. Pearson, M. A. Garcia, S. D. Bader, and A. Hoffmann, Phys. Rev. Lett. **103**, 166601 (2009).
- [29] Z. Wang, H. Liu, H. Jiang, and X. C. Xie, Phys. Rev. B **94**, 035409 (2016).

- [30] See Supplemental Material (which includes Refs. [31–33]) at <http://> for details on the validation tests of our real-space Kubo formula for σ_{SH} ; comparison of θ_{SH} for Au vs. Tl adatoms; and LB formula calculations of R_{NL} and θ_{SH} on additional clean and disordered multiterminal-terminal graphene geometries.
- [31] C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005).
- [32] L. Sheng, D. N. Sheng, C. S. Ting, and F. D. M. Haldane, Phys. Rev. Lett. **95**, 136602 (2005).
- [33] K. Kazymyrenko and X. Waintal, Phys. Rev. B **77**, 115119 (2008); M. Wimmer and K. Richter, J. Comp. Phys. **228**, 8548 (2009).
- [34] C. Weeks, J. Hu, J. Alicea, M. Franz, and R. Wu, Phys. Rev. X **1**, 021001 (2011).
- [35] A. Avsar, J. H. Lee, G. K. Wai Koon, and B. Özyilmaz, 2D Mater. **2**, 044009 (2015).
- [36] A. Weisse, G. Wellein, A. Alvermann, and H. Fehske, Rev. Mod. Phys. **78**, 275 (2006).
- [37] S. Roche and D. Mayou, Phys. Rev. Lett. **79**, 2518 (1997); S. Roche, Phys. Rev. B **59**, 2284 (1999); A. Ferreira and E. R. Mucciolo, Phys. Rev. Lett. **115**, 106601 (2015).
- [38] F. Ortmann and S. Roche, Phys. Rev. Lett. **110**, 086602 (2013); F. Ortmann, N. Leconte, and S. Roche, Phys. Rev. B **91**, 165117 (2015).
- [39] J. H. Garcia, L. Covaci, and T. G. Rappoport, Phys. Rev. Lett. **114**, 116602 (2015).
- [40] T. L. van den Berg, L. Raymond, and A. Verga, Phys. Rev. B. **84**, 245210 (2011); J. H. Garcia and T. G. Rappoport, 2D Mater. **3**, 024007 (2016).
- [41] A. Dyrdal, V. K. Dugaev, and J. Barnas, Phys. Rev. B. **80**, 155444 (2009).
- [42] C. W. Groth, M. Wimmer, A. R. Akhmerov, and X. Waintal, New J. Phys. **16**, 063065 (2014).
- [43] B. K. Nikolić, L. P. Zárbo, and S. Souma, Phys. Rev. B **72**, 075361 (2005); L. Sheng, D. N. Sheng, and C. S. Ting, Phys. Rev. Lett. **94**, 016602 (2005); B. K. Nikolić and R. L. Dragomirova, Semicond. Sci. Tech. **24**, 064006 (2009).
- [44] B. K. Nikolić, L. P. Zárbo, and S. Souma, Phys. Rev. B **73**, 075303 (2006).
- [45] J. Tworzydło, B. Trauzettel, M. Titov, A. Rycerz, and C. W. J. Beenakker, Phys. Rev. Lett. **96**, 246802 (2006); P.-H. Chang, F. Mahfouzi, N. Nagaosa, and B. K. Nikolić, Phys. Rev. B **89**, 195418 (2014).
- [46] M. Titov, EPL **79**, 17004 (2007).